

Informatik Q3 Abels

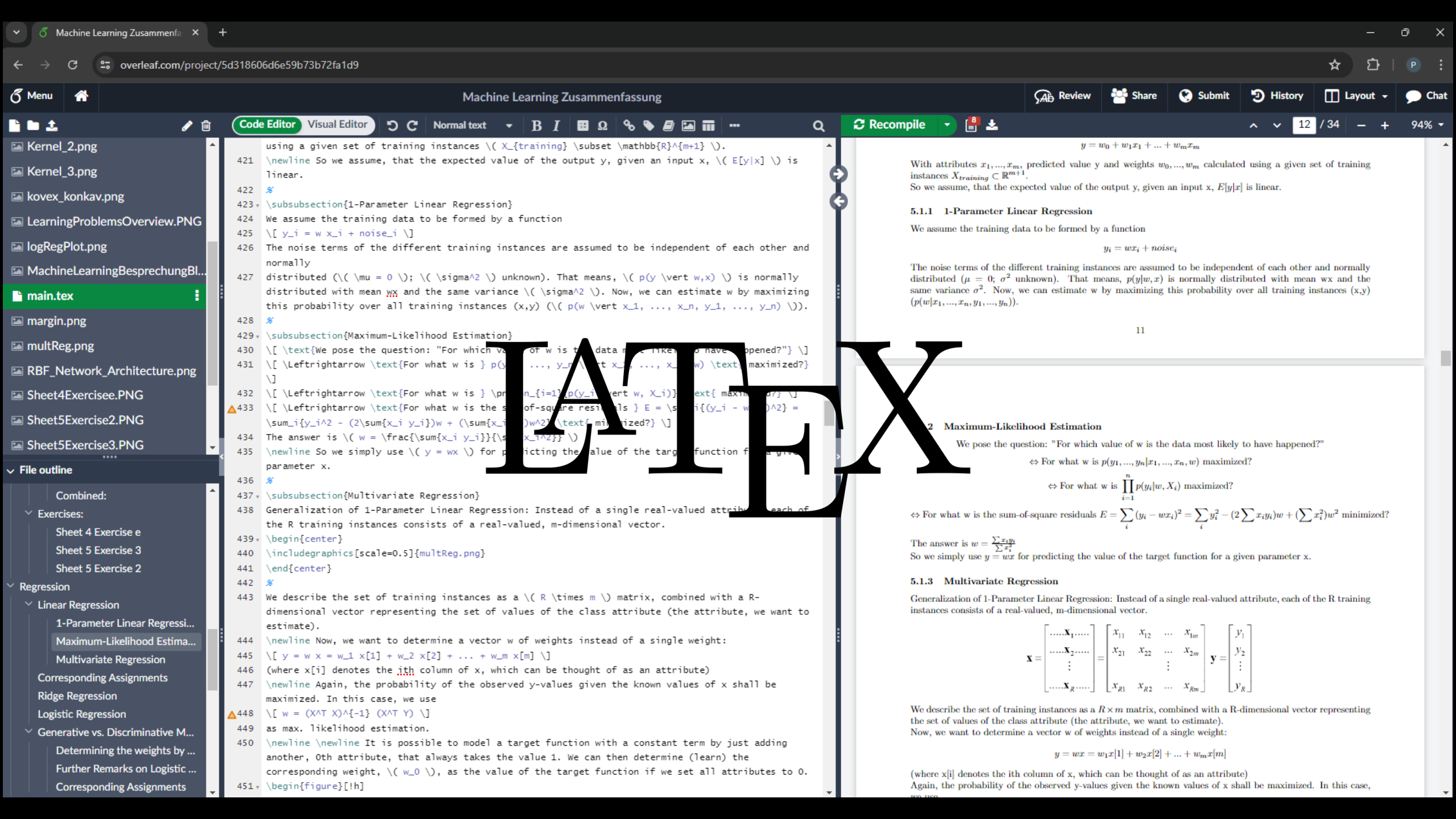


Konzepte und Anwendungen der Theoretischen Informatik



Theoretische Informatik





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using a given set of training instances  $X_{\text{training}} \subset \mathbb{R}^{m+1}$ .
421 \newline So we assume, that the expected value of the output  $y$ , given an input  $x$ ,  $E[y|x]$  is
linear.
422 *
423 \subsubsection{1-Parameter Linear Regression}
424 We assume the training data to be formed by a function
425  $[y_i = w x_i + \text{noise}_i]$ 
426 The noise terms of the different training instances are assumed to be independent of each other and
normally
427 distributed ( $\mu = 0$ ;  $\sigma^2$  unknown). That means,  $p(y | w, x)$  is normally
distributed with mean  $w x$  and the same variance  $\sigma^2$ . Now, we can estimate  $w$  by maximizing
this probability over all training instances  $(x, y)$  ( $p(w | x_1, \dots, x_n, y_1, \dots, y_n)$ ).
428 *
429 \subsubsection{Maximum-Likelihood Estimation}
430  $[\text{We pose the question: "For which value of } w \text{ is the data most likely to have happened?"}]$ 
431  $[\text{For what } w \text{ is } p(y_1, \dots, y_n | x_1, \dots, x_n, w) \text{ maximized?}]$ 
432  $[\text{For what } w \text{ is } \prod_{i=1}^n p(y_i | w, x_i) \text{ maximized?}]$ 
433  $[\text{For what } w \text{ is the sum-of-square residuals } E = \sum_i (y_i - wx_i)^2 =$ 
 $\sum_i y_i^2 - 2(\sum_i x_i y_i)w + (\sum_i x_i^2)w^2 \text{ minimized?}]$ 
434 The answer is  $w = \frac{\sum_i x_i y_i}{\sum_i x_i^2}$ 
435 \newline So we simply use  $y = wx$  for predicting the value of the target function for a given
parameter  $x$ .
436 *
437 \subsubsection{Multivariate Regression}
438 Generalization of 1-Parameter Linear Regression: Instead of a single real-valued attribute each of
the  $R$  training instances consists of a real-valued,  $m$ -dimensional vector.
439 \begin{center}
440 \includegraphics[scale=0.5]{multReg.png}
441 \end{center}
442 *
443 We describe the set of training instances as a  $(R \times m)$  matrix, combined with a  $R$ -
dimensional vector representing the set of values of the class attribute (the attribute, we want to
estimate).
444 \newline Now, we want to determine a vector  $w$  of weights instead of a single weight:
445  $[y = w x = w_1 x[1] + w_2 x[2] + \dots + w_m x[m]]$ 
446 (where  $x[i]$  denotes the  $i$ th column of  $x$ , which can be thought of as an attribute)
447 \newline Again, the probability of the observed  $y$ -values given the known values of  $x$  shall be
maximized. In this case, we use
448  $[w = (X^T X)^{-1} (X^T Y)]$ 
449 as max. likelihood estimation.
450 \newline \newline It is possible to model a target function with a constant term by just adding
another, 0th attribute, that always takes the value 1. We can then determine (learn) the
corresponding weight,  $w_0$ , as the value of the target function if we set all attributes to 0.
451 \begin{figure}[!h]

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LATEX

$y = w_0 + w_1 x_1 + \dots + w_m x_m$

With attributes x_1, \dots, x_m , predicted value y and weights w_0, \dots, w_m calculated using a given set of training instances $X_{\text{training}} \subset \mathbb{R}^{m+1}$. So we assume, that the expected value of the output y , given an input x , $E[y|x]$ is linear.

5.1.1 1-Parameter Linear Regression

We assume the training data to be formed by a function

$$y_i = wx_i + \text{noise}_i$$

The noise terms of the different training instances are assumed to be independent of each other and normally distributed ($\mu = 0$; σ^2 unknown). That means, $p(y|w, x)$ is normally distributed with mean $w x$ and the same variance σ^2 . Now, we can estimate w by maximizing this probability over all training instances (x, y) ($p(w | x_1, \dots, x_n, y_1, \dots, y_n)$).

11

5.2 Maximum-Likelihood Estimation

We pose the question: "For which value of w is the data most likely to have happened?"

\Leftrightarrow For what w is $p(y_1, \dots, y_n | x_1, \dots, x_n, w)$ maximized?

\Leftrightarrow For what w is $\prod_{i=1}^n p(y_i | w, X_i)$ maximized?

\Leftrightarrow For what w is the sum-of-square residuals $E = \sum_i (y_i - wx_i)^2 = \sum_i y_i^2 - 2(\sum_i x_i y_i)w + (\sum_i x_i^2)w^2$ minimized?

The answer is $w = \frac{\sum_i x_i y_i}{\sum_i x_i^2}$

So we simply use $y = wx$ for predicting the value of the target function for a given parameter x .

5.1.3 Multivariate Regression

Generalization of 1-Parameter Linear Regression: Instead of a single real-valued attribute, each of the R training instances consists of a real-valued, m -dimensional vector.

$$X = \begin{bmatrix} \dots & X_1 & \dots \\ \dots & X_2 & \dots \\ \vdots & \vdots & \vdots \\ \dots & X_R & \dots \end{bmatrix} = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1m} \\ x_{21} & x_{22} & \dots & x_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ x_{R1} & x_{R2} & \dots & x_{Rm} \end{bmatrix} \quad y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_R \end{bmatrix}$$

We describe the set of training instances as a $R \times m$ matrix, combined with a R -dimensional vector representing the set of values of the class attribute (the attribute, we want to estimate). Now, we want to determine a vector w of weights instead of a single weight:

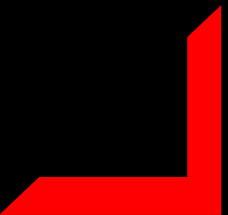
$$y = wx = w_1 x[1] + w_2 x[2] + \dots + w_m x[m]$$

(where $x[i]$ denotes the i th column of x , which can be thought of as an attribute)

Again, the probability of the observed y -values given the known values of x shall be maximized. In this case,



Tagebucheintrag





Wochenübung

- Schaffe dein persönliches Setup zur Theoretischen Informatik.
- Schaue dir meine Empfehlungen an:
 - Overleaf:
<https://www.overleaf.com/>
 - Overleaf Tutorial:
<https://www.youtube.com/watch?v=Jp0IPj2-DQA&list=PLHXZ9OQGMqxcWWkx2DMnQmj5os2X5ZR73>
 - Latex Tutorial:
<https://www.youtube.com/watch?v=hRwUjJYeHjw&list=PLuyjaM3Uz-oOS7zcMFaROwrg83KBR1Sui>
 - Inhalte auf Youtube:
https://www.youtube.com/watch?v=5jFKI_G5QWg&list=PLG_1tsKrsKVO2ANHX68UbrNgt7gZuH37H => NICHT ALLES!
 - Inhalte spielerisch lernen:
<https://inf-schule.de/automaten-sprachen> => NICHT ALLES!

